DUAL LIGHT-CONE MODEL PREDICTIONS FOR TOTAL AND INCLUSIVE e^+e^- ANNIHILATION

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Predictions of a recently proposed dual scaling model for the total and inclusive e^+e^- cross section are discussed taking particular interest in preasymptotic corrections. Good agreement with available data is found.

Recently, we have presented a dual light-cone model for deep inelastic electroproduction and annihilation [1] which led to an explicit ansatz for the structure functions $F_2(x)$ and $\bar{F}_2(x)$. In particular, we found a generalized Gribov–Lipatov reciprocity relation [2] which connects $F_2(x)$ and $\bar{F}_2(x)$ in their physical regions so that the continuation to the deep inelastic annihilation region becomes exceedingly simple. Once our model has been tested in the scattering region, we thus can predict the annihilation structure function $\bar{F}_2(x)$ on firm grounds of the electroproduction data.

Preliminary SPEAR results show [3] that, at present energies ($q^2 \lesssim 25\ \mathrm{GeV}^2$), the inclusive cross section does not scale in the annihilation region except near $x \approx 1$ which is in sharp contrast to the experimental finding of early scaling in deep inelastic electroproduction and raises the question, whether scaling has not yet been reached or even is broken in the timelike region. In order to decide this question and before one draws any conclusion against Bjorken scaling for positive q^2, one has to examine closely what current scaling models predict in the annihilation region and to look for preasymptotic corrections which we have good reasons to believe play an important role at SPEAR energies. This may reveal that the SPEAR results do not contradict asymptotic scaling though (contrary to may augurs), perhaps, they are not the experimentum crucis for probing parton structures.

In this paper we shall discuss the predictions of our model [1] for total and inclusive e^+e^- annihilation taking particular interest in nonleading (scale breaking) contributions. We shall see that the preliminary SPEAR data can be well described in terms of this (scaling) model.

The (scattering) structure function $F_2(x)$ was given by [1] [‡1] ($x = -q^2/2\nu$)

$$F_2(x) = N x^{-\alpha(0)+1}(1-x)^{2c_1+1} \int_{-1}^{+1} \mathrm{d}\beta \left(\frac{1-\beta'^2}{4}\right)^{c_1} \times \left(\frac{(1+x)^2-(1-x)^2\beta'^2}{4}\right)^{-c_1+c_1'+\alpha(0)-2}, \tag{1}$$

where c_1 and c_1' are determined by the asymptotic behavior of the (electromagnetic) target form factor and the $(2^+)\to(1^-)$ transition form factor respectively. The normalization of the structure function (i.e., N) is provided by the Adler sum rule ($\alpha(0)<1$)

$$\int_0^1 \frac{\mathrm{d}x}{x} F_2(x) = 1\,, \tag{2}$$

being a consequences of the current algebra constraints wich actually led to scaling. The (annihilation) structure function $\bar{F}_2(x)$ could most simply be expressed in terms of $F_2(x)$ by means of the reciprocity relation (which

‡1 We shall not take into account corrections on the daughter level which were necessary to obtain a selfconsistent fixed pole residue (for details see ref. [1]). We have convinced ourselves in the case of nucleon Compton scattering that these corrections are indeed negligible. On the other hand, the model should not be taken too seriously on this level.

holds for arbitrary c_1)

$$\bar{F}_2(x) = x^{2c_1'-1} F_2(1/x)\,. \tag{3}$$

In the following we shall set $c_1' = 2$ in accordance with other models [2,4]. We furthermore take $c_1 = 0$ for mesons (monopole form factor) and $c_1 = 1$ for nucleons (dipole form factor) respectively.

Assuming SU(3) (exchange degeneracy) nonexoticity in all channels and the photon being a U-spin scalar, we obtain for the nondiffractive part of the pseudoscalar meson octet the (physical) scaling functions

$$F_2^{\pi^\pm,\pi^0,K^\pm}(x) = \tfrac{5}{2} F_2^{K^0,\bar{K}^0}(x) = \tfrac{5}{3} F_2^{\eta}(x) = \tfrac{5}{9} F_2(x)\,, \tag{4}$$

and similarly for the annihilation structure functions. In the case of the baryon (antibaryon) octet we have to allow for a 1 0 ($1\,\bar{0}$) representation in the baryonic channels. If we assume that the decuplet does not contribute in the scaling region, we would get the analogue of eq. (4). This might be justified for $x \approx 1$ since the $\Delta(1236) \to$ N transition form factor shows a slightly faster decrease [5] than the nucleon form factor which suggest a suppression near $x = 1$ via the Drell–Yan relation. But, in general, there is no doubt that the decuplet contributes to the scaling functions [6]. Another solution (which treats N and Δ on the same footing) would be the SU(6)/quark model result [7]

$$F_2^{p,\Sigma^+}(x) = \tfrac{3}{2} F_2^{n,\Sigma^0,\Xi^0,\Lambda}(x) = 3 F_2^{\Sigma^-,\Xi^-}(x) = F_2(x)\,. \tag{5}$$

A detailed analysis of the various symmetry aspects allowing for a different threshold behavior of the decuplet contribution which also explains the rather small $F_2^n(x)/F_2^p(x)$ ratio will be given in a forthcoming paper [8].

The pomeron contribution cannot be integrated in the duality scheme so far developed, but has to be added by hand. It is tempting to assume the same ansatz (eq. (1)) for the diffractive terms as for the nondiffractive term as for the nondiffractive part (here $\alpha(0) = 1$ of course). This can be motivated in our model (having nonlinear trajectories) by replacing the s-channel trajectory by some background trajectory without resonances being dual to the pomeron in accordance with the Harari–Freund conjecture. Here, the threshold behavior (i.e., c_1) is, however, no longer determined by the asymptotic behavior of any form factor as in the nondiffractive case. But following the general belief that the background corresponds (at least) to a four and five quark assignment we conclude [9] $c_1 = 2$

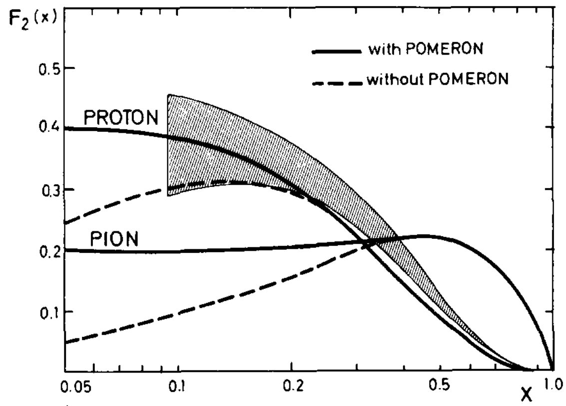

Fig. 1. The nucleon and pion scaling functions. The shaded area corresponds to the SLAC electroproduction data.

and $c_1 = 3$ for mesons and baryons respectively. Hence, the pomeron contribution is strongly suppressed near threshold compared to the nondiffractive part which seems to be supported by comparison of neutrino and electroproduction experiments [10].

In the following we take the SU(6) solution for the nucleon octet. We furthermore assume that the pomeron be a SU(3) singlet. Then, the normalization of the pomeron contribution is the only free parameter. The intercept of the Regge trajectory is taken to be $\alpha(0) = 0.3$ (what would come out if the trajectory would rise linearly up to the A_2, f resonance with $\alpha'(0) = 1\ \mathrm{GeV}^{-2}$). In fig. 1 we have drawn $F_2^p(x)$ taking $N_{\mathrm{pomeron}} = 2$, which gives a good fit to the data [‡2]. Also indicated is the pomeron contribution. This is significant only at small x as is to be expected.

In the case of the meson octet there is no ambiguity as far as the SU(3) structure is concerned. Here the pomeron coupling is via factorization determined by the ratio $\sigma_{\mathrm{tot}}^{\pi p}/\sigma_{\mathrm{tot}}^{pp} = 2/3$ (note that the $x \to 0$ limit of the scaling functions does not depend on c_1). The resulting scaling function is shown in fig. 1. Due to the different threshold factor, $F_2^\pi(x)$ increases much faster near threshold than the proton scaling function. The pomeron contribution again is negligible for $x \approx 1$ but is considerably larger for medium x ($x \approx 0.2$) than in the case of the proton.

The annihilation structure functions can be deduced from fig. 1 employing the reciprocity relation (3).

‡2 For $\alpha(0) = 0.5$ we would have to allow a higher normalization of the scaling function to obtain a fit of comparable quality.

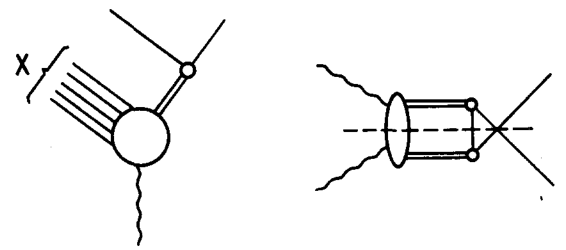

Fig. 2. The resonance production diagrams.

For large x we have $\bar{F}_2(x) \sim x^{\alpha(0)+1}$ for the nondiffractive term and $\bar{F}_2(x) \sim x^3$ for the pomeron contribution.

So far we have concentrated on the pion and nucleon (octet) structure functions only. But any other particle h which gives rise to scaling in the deep inelastic scattering region, will contribute a portion to the large-q^2 total inclusive e^+e^- cross section as well. If we had exact SU(6), e.g., we would get 35/8 times as many signals as if there were only pions. In the case that the particle h is unstable (broken SU(6)) this gives of course, a contribution to the inclusive spectrum of its decay products as shown in fig. 2a.

In the imaginary part of the (annihilation) Compton amplitude this process (fig. 2a) corresponds to the double h exchange diagram as drawn in fig. 2b for ρ decaying into two pions. As is well known, this diagram ($q^2 > 0$) gives rise to anomalous singularities [‡3] which certainly are not included in the pion structure function $\bar{F}_2^{\pi}(x)$ so far considered. The reason being that our model only accounts for normal threshold singularities. Hence, it is plausible to add these anomalous singularity contributions to the pion (annihilation) structure function $\bar{F}_2^{\pi}(x)$, whereas the normal threshold part can be thought of being already included in our model.

In the scaling limit ($q^2 \to \infty$) the ρ production diagram gives, in the zero width approximation, rise to the (anomalous) cut contribution [11] (supplementary to the normal threshold pion scaling function $\bar{F}_2^{\pi}(x)$):

$$\frac{3m_\rho}{p^\rho_{\mathrm{cm}}} \int_1^x \mathrm{d}\eta\, \eta \bar{F}_2^\rho \left(\frac{x}{\eta}\right) \theta \left(m_\rho^2 + m_\pi^2 \frac{\eta^2}{1-\eta}\right), \tag{6}$$

‡3 For a thorough discussion of this diagram in respect to the scaling functions, see ref. [11].

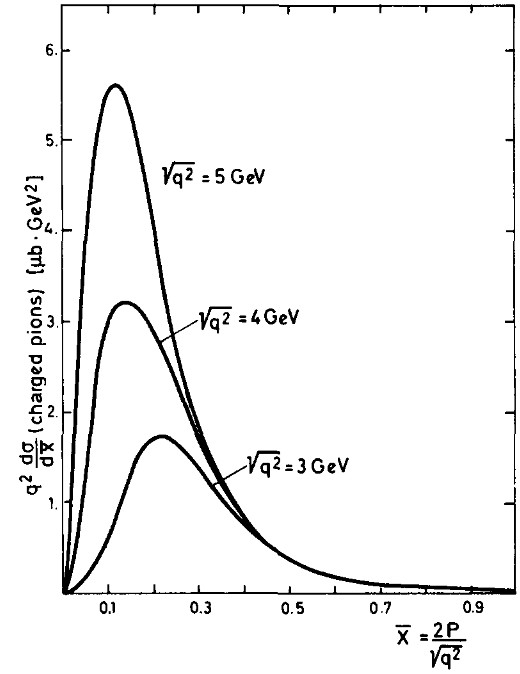

Fig. 3. Prediction of the charged pion inclusive cross section.

and similarly for any other resonance (if, say the ρ would decay into a pion and a different particle of mass $\bar{m}$, formula (6) had to be devided by 2 and the θ function be replaced by $\theta(m_\rho^2 - m_\pi^2\eta + \bar{m}^2\eta/(1-\eta))$. In this approximation the ρ is forced to be on the mass shell so that the (spin averaged) ρ structure function appears under the integral. For $\bar{F}_2^\rho(x)$ (as well as for the other SU(3) partners) we make the same ansatz as for the pion structure function (apart from a possibly different c_1) which leads to the analogue of eq. (4). It is tempting to assume $c_1 = 0$ also (for the nondiffractive part) as one would expect from SU(6). Other resonances like, e.g., the A_1 which prominently decay into three and more particles can be handled in a similar way. In case of the A_1, e.g., we would have another pion ring in fig. 2b due to the cascade $A_1 \to \rho\pi \to 3\pi$.

For finite q^2 preasymptotic corrections resulting from kinematical facators become very important in expression (6), especially for large x, even at the highest SPEAR energy ($q^2 = 25\ \mathrm{GeV}^2$). This happens because the dominant contribution to the integral (6) comes from large arguments of $\bar{F}_2^\rho(x)$ (remember

$\bar{F}_2^{\rho}(x) \sim x^{\alpha(0)+2}$), whereas the somewhat higher resonance mass gives rise to threshold factors which sensitively cut off this region. Thus, in order to make substantial predictions at SPEAR energies, we definitely have to include these effects. In the following we shall quote the results only, but will give a detailed analysis of these corrections in a more extensive paper [8].

We shall now assume the validity of the Callan–Gross relation in order to define the scaling function $\bar{F}_1(x)$ (in ref. [1] we have argued that the Calan–Gross relation should hold irrespective of the spin of the constituents). In the scaling limit ($q^2 \to \infty$) the Callan–Gross relation also is maintained for the cut contributions (6) which then leads to the inclusive cross section [‡4] for $e^+e^- \to h + X$:

$$q^2 \frac{d\sigma^h}{d\bar{x}} = \pi\alpha^2 \frac{\beta_h^2}{x^2}\left(1-\frac{\beta_h^2}{3}\right)\bar{F}_2^h(x) \,, \tag{7}$$

where $\bar{x} = 2p/\sqrt{q^2}$ and $\beta_h^2 = 1 - 4m_h^2 x^2/q^2$. For finite q^2 (even $q^2 = 25\ \mathrm{GeV}^2$), however, the Callan–Gross relation no longer holds for the resonance terms [8] but threshold factors become involved which have a considerable effect on eq. (7).

In fig. 3 we have drawn our predication for the (total) charged inclusive $e^+e^- \to \pi + X$ cross section taking into account (besides the pion contribution) the lowest lying resonances δ, ρ, A_1 and B and their octet partners [12] and commonly assuming $c_1 = 0$ which may be justified upon symmetry arguments. In the small $\bar{x}$ region (large x) the resonance terms provide by far the dominant contribution, whereas they are negligible for $\bar{x} \gtrsim 0.5$. Hence, we are not surprised that scaling breaks down at SPEAR energies for [‡5] $\bar{x} \lesssim 0.5$. In fact, the shape and order of magnitude of the predicted inclusive cross section is in agreement with the preliminary SPEAR data [3] [‡6]. A precise prediction [‡7] depends, however, on the effect of the higher (excited) resonances like A_2, f, etc. although we believe that their contribution is small at SPEAR energies due to the increasing mass and a likely more suppressing Drell–Yan threshold factor.

We also have looked at the angular distribution of the pion and find a substantial deviation from the asymptotic form $\sim 1 + \cos^2\theta$. At $q^2 = 25\ \mathrm{GeV}^2$ the pion distribution is absolutely flat in the region $\bar{x} \lesssim 0.3$ (which accounts for most of the events) and beyond that region gradually turns over into the asymptotic form. For increasing (decreasing) q^2 the boundary between these two regions is shifted towards lower (higher) $\bar{x}$ being consistent with the asymptotic distribution.

The total cross section is given by the energy conservation sum rule

$$\sigma(e^+e^- \to \text{hadrons}) = \tfrac{1}{2}\sum_h \int_1^{\sqrt{q^2}/2m_h} \frac{dx}{x}\frac{d\sigma^h}{dx} \,, \tag{8}$$

where the sum is over all participating hadrons. We now assume that all the available energy is carried away by pions and kaons and calculate the total cross section from the inclusive spectrum of these particles (note that in our model the neutrals are produced with the same strength as charged particles). The kaons are found to contribute roughly 15% to the total cross section. The results is shown in fig. 4 and compared with the world's data. At higher q^2 the predicted cross section is substantially smaller than the experimental cross section which is somewhat surprising. We would have expected that both roughly agree since the predicted inclusive cross section is consistent with the preliminary SPEAR data [3]. In order to explain this (energv crisis [13]) a large fraction of the energy has to be carried away by neutrals or by particles other than pions and kaons (perhaps baryons). The general feature that the ratio R increases with energy, however, is well reproduced.

So far we have assumed perfect scaling for the structure functions. Neglecting curvature of the Regge trajectories, we obtain that the preasymptotic corrections to our scaling functions can, in first order, be accomodated by rescaling of the variable x:

$$x \to x' = x\,\frac{1 + a/q^2}{1 - bx/q^2} \,,$$
$$a = (\alpha(0) + \tfrac{1}{2})/\alpha'(0) \,, \quad b = (\tfrac{1}{2} + c_1 - \alpha(0))/\alpha'(0) \,, \tag{9}$$

‡4 The scaling function $\bar{F}_2^h(x)$ is always understood spin averaged. If the spin of particle h is not detected, eq. (7) has to be multiplied by $2J+1$.

‡5 Had we included kaons also, we would have obtained an energy splitting up to larger values of $\bar{x}$.

‡6 Note that the kaons account for 10% of the data, whereas the nucleon contribution is found to be very small.

‡7 There is also the possibility that the normalization of the scaling functions deviates from that prescribed by the Adler sum rule due to SU(3) and exchange degeneracy breaking effects.

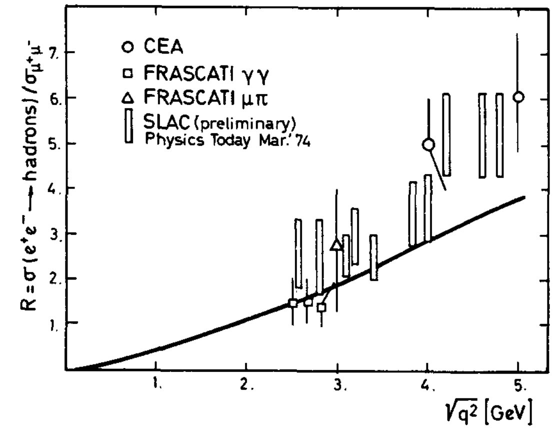

Fig. 4. Predicted total cross section compared to the world's data.

where α is meant to be the trajectory in the photon channel (note that a and b do not depend on the external masses). The variable x' has long been advocated by Bloom and Gilman [6] (here $a = 0$) and by Rittenberg and Rubinstein [14] and for electroproduction allows the concept of scaling to be extended down to very low values of q^2. For proton targets we would predict ($\alpha(0) = 0.3$) $a = 0.8\ \mathrm{GeV}^2$ and $b = 1.2\ \mathrm{GeV}^2$ being in not too bad agreement with experiment [15]. In the annihilation region the substitution (9) leads to higher effective x values and, hence, to higher cross sections because of the singular behavior of the scaling functions at $x \to \infty$. The resonance contributions which account for most of the cross section will, however, not very much be effected by these corrections since they are confined to smaller x in the resonance structure functions.

In our model (s, u) terms do not contribute to the current algebra fixed pole and, hence, do not survive in the scaling limit. However, they may give rise to nonnegligible contributions at finite q^2, especially in the very large x region as has been emphasized by Satz [16]. From SU(3) and nonexoticity we derive the general relation

$$\begin{aligned} \nu \bar{W}_2^{\pi^\pm, \mathrm{K}^\pm}(s,u) &= -\tfrac{4}{5}\, \nu \bar{W}_2^{\pi^0}(s,u) \\ &= -\tfrac{4}{2}\, \nu \bar{W}_2^{\mathrm{K}^0, \bar{\mathrm{K}}^0}(s,u) = -\tfrac{4}{3}\, \nu \bar{W}_2^{\eta}(s,u)\,. \end{aligned} \tag{10}$$

We see that neutral particles contribute with a sign opposite to charged particles. This sets, of course, an upper limit to the (s, u) terms because the total structure function has to be positive definite. Hence, we cannot expect that the (s, u) terms contribute much to the total cross section since most of the contributions cancel out. But they might have a substantial effect on the neutral to charged ratio and lead a way out of the energy crisis [13].

The most challenging question now is that of the asymptotic value of R. In order to draw any conclusions from our model we have to include the higher mesons and the baryons which might give a sensible contribution at very high energies. For a finite number of resonances (being in the spirit of our model with nonlinear trajectories) R will tend to a constant what we would expect if QED continues to hold at very small distances.

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